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NEW ANALYTIC SOLUTIONS OF THE WAVE EQUATION AND THE PROBLEM OF DIFFRACTION

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An earlier paper [1] gave the exact solutions for cylindrical and spherical waves, which made possible the solution of the problem of diffraction of waves due to a three-dimensional and a plane source. In the present paper the class of exact solutions is expanded significantly. The problem of diffraction of a wave due to a plane source by a semi-infinite plate is solved in a finite form.

1. We know [1] that if a solution of the wave equation

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}-\frac{\partial^2\Phi}{\partial t^2}=0 \quad (1.1)$$

is homogeneous in t and $r=\sqrt{x^2+y^2}$ of degree $-{}^1/_2$ and has the form $\Phi_{-1/2}(t, r, \theta)$, then $\Phi_{-1/2}(t+\alpha(t^2-r^2), r, \theta)$, where $\alpha=\text{const}$ and $\theta=\text{arctg}\,(y/x)$, also satisfies (1.1). On the other hand, the relation connecting the homogeneous solutions of the wave equation which have different degrees, is well known. In particular, if Φ_0 and Φ_n are solutions of (1.1) homogeneous in t and r of degrees 0 and n, respectively, and such that $(\Phi_n/t^n)|_{t=r}=\Phi_0|_{t=r}$, then they are connected by the following relation [2, 3]:

$$\Phi_n=\frac{(-1)^n 2^n n!}{(2n)!}(t^2-r^2)^{n+1/2}\frac{\partial^n}{\partial t^n}\frac{\Phi_0(r,t,\theta)}{\sqrt{t^2-r^2}} \quad (1.2)$$

Let us now set, in a purely formal way, the sum of the solutions of the wave equation, using the relation (1.2)

$$\Phi = \sqrt{t^2 - r^2} \sum_{n=0}^{\infty} \frac{\alpha^n (t^2 - r^2)^n}{n!} \frac{\partial^n}{\partial t^n} \frac{\Phi_0 (t, r, \theta)}{\sqrt{t^2 - r^2}}$$

This infinite sum can be replaced by the expression

$$\Phi = \frac{\Phi_0 (t + \alpha (t^2 - r^2), r, \theta)}{\sqrt{1 + 2\alpha t + \alpha^2 (t^2 - r^2)}} \tag{1.3}$$

Thus, in addition to the Filippov theorem [1], we have the following assertion: if Φ_0 (t, r, θ) is a solution of (1.1) homogeneous in t and r and of degree 0, then the expression (1.3) is also a solution of the wave equation.

Using both results, we can formulate the following problem: to find a function η (x, y, t) such that the function

$$\Phi = \eta (x, y, t)\ \Phi_\beta (X, Y, T) \tag{1.4}$$

satisfies the wave equation (1.1), here Φ_β is a solution of the wave equation written in the coordinates X, Y and T where $X = x$, $Y = y$ and $T = t + \alpha (t^2 - r^2)$, homogeneous in X, Y and T and of degree β. Substituting the product (1.4) into (1.1) and making use of the relations

$$\frac{\partial \Phi_\beta}{\partial x} = \frac{\partial \Phi_\beta}{\partial X} - 2\alpha x \frac{\partial \Phi_\beta}{\partial T} \quad \text{etc.} \quad \frac{\partial^2 \Phi_\beta}{\partial X^2} + \frac{\partial^2 \Phi_\beta}{\partial Y^2} - \frac{\partial^2 \Phi_\beta}{\partial T^2} = 0$$

we obtain

$$\Phi_\beta \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} - \frac{\partial^2 \eta}{\partial t^2}\right) + 2 \left\{\frac{\partial \eta}{\partial x} \left(\frac{\partial \Phi_\beta}{\partial X} - 2\alpha x \frac{\partial \Phi_\beta}{\partial T}\right) + \frac{\partial \eta}{\partial y} \left(\frac{\partial \Phi_\beta}{\partial Y} - 2\alpha y \frac{\partial \Phi_\beta}{\partial T}\right) - (1 + 2\alpha t) \frac{\partial \eta}{\partial t} \frac{\partial \Phi_\beta}{\partial T}\right\} + \eta \left\{- 4\alpha T \frac{\partial^2 \Phi_\beta}{\partial T^2} - 6\alpha \frac{\partial \Phi_\beta}{\partial T} - 4\alpha \frac{\partial}{\partial T} \left(X \frac{\partial \Phi_\beta}{\partial X} + Y \frac{\partial \Phi_\beta}{\partial Y}\right)\right\} = 0 \tag{1.5}$$

since Φ_β is a homogeneous function, we have

$$X \frac{\partial \Phi_\beta}{\partial X} + Y \frac{\partial \Phi_\beta}{\partial Y} + T \frac{\partial \Phi_\beta}{\partial T} = \beta \Phi_\beta$$

Therefore the expression (1.5) can be written in the form

$$\Phi_\beta \left\{\frac{\partial^2 \eta}{\partial x^2} + \frac{2\beta}{y} \frac{\partial \eta}{\partial y} + \frac{\partial^2 \eta}{\partial y^2} - \frac{\partial^2 \eta}{\partial t^2}\right\} + 2 \left\{\frac{\partial \eta}{\partial x} - \frac{x}{y} \frac{\partial \eta}{\partial y}\right\} \frac{\partial \Phi_\beta}{\partial X} - 2 \left\{2\alpha x \frac{\partial \eta}{\partial x} + \left(\frac{T}{y} + 2\alpha y\right) \frac{\partial \eta}{\partial y} + (1 + 2\alpha t) \frac{\partial \eta}{\partial t} + \alpha (2\beta + 1) \eta\right\} \frac{\partial \Phi_\beta}{\partial T} = 0 \tag{1.6}$$

We shall require the homogeneous function Φ_β to be arbitrary, and in that case the expressions within the curly brackets in (1.6) must each be separately made equal to zero. We obtain three equations for determining η, and passing in these equations to the new variables $2\alpha x = \mu$, $2\alpha y = \nu$ and $2\alpha t = \tau$, we have

$$\frac{\partial^2 \eta}{\partial \mu^2} + \frac{2\beta}{\nu} \frac{\partial \eta}{\partial \nu} + \frac{\partial^2 \eta}{\partial \nu^2} - \frac{\partial^2 \eta}{\partial \tau^2} = 0, \quad \frac{\partial \eta}{\partial \mu} - \frac{\mu}{\nu} \frac{\partial \eta}{\partial \nu} = 0 \tag{1.7}$$

$$\mu \frac{\partial \eta}{\partial \mu} + \frac{1}{\nu} \left[\tau + \frac{1}{2} (\tau^2 - \rho^2) + \nu^2\right] \frac{\partial \eta}{\partial \nu} + (1 + \tau) \frac{\partial \eta}{\partial \tau} + \left(\beta + \frac{1}{2}\right) \eta = 0 \quad (\rho^2 = \mu^2 + \nu^2)$$

The second equation of (1.7) shows that η depends on ρ and τ only (it is independent of $\theta = \operatorname{arctg}(\nu / \mu)$). We then have $\partial\eta / \partial\nu = \sin\theta, \partial\eta / \partial\rho\, \partial\eta / \partial\mu = \cos\theta\, \partial\eta/\partial\rho$ and the remaining equations of (1.7) become

$$\frac{\partial^2\eta}{\partial\rho^2} + \frac{2\beta+1}{\rho}\frac{\partial\eta}{\partial\rho} - \frac{\partial^2\eta}{\partial\tau^2} = 0 \tag{1.8}$$

$$\frac{1}{\rho}\left[\tau + \frac{1}{2}(\tau^2 - \rho^2)\right]\frac{\partial\eta}{\partial\rho} + (1+\tau)\frac{\partial\eta}{\partial\tau} + \left(\beta + \frac{1}{2}\right)\eta = 0$$

The second equation of (1.8) has the following solution:

$$\eta = \frac{f(\xi)}{(\tau+1)^{\beta+1/2}}, \qquad \xi = \frac{\tau^2 - \rho^2}{\tau + 1}$$

Substituting this expression into the first equation of (1.8) we obtain

$$(\xi^2 + 4\xi) f'' + (2\beta + 3)(2 + \xi) f' + {}^1/_4 (2\beta + 1)(2\beta + 3) f = 0$$

which has the following solution:

$$f(\xi) = C_1 / \xi^{\beta+1/2} + C_2 / (\xi + 4)^{\beta+1/2}$$

where C_1 and C_2 are constants.

Let us select a solution which has no singularities at $\tau = \rho$. Then the solution of the problem given above is

$$\Phi = \frac{\Phi_\beta\,[x, y, t + \alpha(t^2 - r^2)]}{[(1+\alpha t)^2 - \alpha^2 r^2]^{\beta+1/2}} \tag{1.9}$$

The result given in [1] and the relation (1.3) follow from (1.9) as particular cases.

2. Filippov also mentions in [1] the method of constructing the solution of the problem of diffraction of a spherical wave due to a source , by a wedge. This case can also be similarly generalized to an arbitrary function homogeneous in t and $r = \sqrt{x^2+y^2}$ of any degree, provided that this function is a solution of (1.1). As before, we formulate the following problem: to find a function $\eta(x, y, z, t)$ such that the function $\Phi = \eta\Phi_\beta(T, P, \Theta)$ satisfies the wave equation

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} + \frac{\partial^2\Phi}{\partial z^2} - \frac{\partial^2\Phi}{\partial t^2} = 0$$

provided that Φ_β is of degree β , homogeneous in T and P and that it satisfies the equation

$$\frac{\partial^2\Phi_\beta}{\partial P^2} + \frac{1}{P}\frac{\partial\Phi_\beta}{\partial P} + \frac{1}{P^2}\frac{\partial^2\Phi_\beta}{\partial\Theta^2} - \frac{\partial^2\Phi_\beta}{\partial T^2} = 0$$

$$(P = r,\ \Theta = \theta,\ T = t + \alpha(t^2 - r^2 - z^2))$$

We omit the cumbersome manipulations analogous to those given in Sect. 1 and write the equations which the function η must satisfy:

$$\frac{\partial\eta}{\partial\theta} = 0, \quad \frac{\partial^2\eta}{\partial\rho^2} + \frac{2\beta+1}{\rho}\frac{\partial\eta}{\partial\rho} + \frac{\partial^2\eta}{\partial\zeta^2} - \frac{\partial^2\eta}{\partial\tau^2} = 0 \tag{2.1}$$

$$\frac{1}{2\rho}(2\tau + \tau^2 + \rho^2 - \zeta^2)\frac{\partial\eta}{\partial\rho} + \zeta\frac{\partial\eta}{\partial\zeta} + (1+\tau)\frac{\partial\eta}{\partial\tau} + (\beta+1)\eta = 0$$

$$(\rho = 2\alpha r, \zeta = 2\alpha z, \tau = 2\alpha t)$$

The last equation of (2.1) shows that

$$\eta=\frac{W(\xi,\lambda)}{(\tau+1)^{\beta+1}},\quad \xi=\frac{\tau^2-\zeta^2-\rho^2}{\tau+1},\quad \lambda=\frac{\zeta}{\tau+1}$$

Substituting this into the second equation of (2.1) and introducing a new variable $\sigma = (\xi + 2)/2$, we obtain

$$(1-\lambda^2)\frac{\partial^2 W}{\partial\lambda^2}+(1-\sigma^2)\frac{\partial^2 W}{\partial\sigma^2}-2\lambda\sigma\frac{\partial^2 W}{\partial\lambda\partial\sigma}-$$
$$2(\beta+2)\lambda\frac{\partial W}{\partial\lambda}-2(\beta+2)\sigma\frac{\partial W}{\partial\sigma}-(\beta+1)(\beta+2)W=0 \quad (2.2)$$

Thus, compared with the problem of Sect. 1, the present problem has a wider class of solutions which now depend on two variables. Without going into a detailed analysis of (2.2) we note that it has been studied extensively. In particular, when $\beta = -1$, the equation (2.2) reduces to the Laplace's equation in R and ψ, where

$$R=\frac{1}{\sqrt{\sigma^2+\lambda^2}}-\sqrt{\frac{1}{\sigma^2+\lambda^2}-1},\quad \psi=\operatorname{arctg}\left(\frac{\lambda}{\sigma}\right)$$

Let us find a solution independent of λ which will be required in what follows. This solution is

$$W = C_1 / \xi^{\beta+1} + C_2 / (\xi + 4)^{\beta+1}$$

Selecting a solution which has no singularities at $\xi = 0$ we obtain the following particular solution of the above problem:

$$\Phi=\frac{\Phi_\beta\,[t+\alpha(t^2-r^2-z^2),\,r,\,\theta]}{[(\alpha t+1)^2-\alpha^2z^2-\alpha^2r^2]^{\beta+1}} \quad (2.3)$$

Omitting the derivation we show another result obtained by analogous transformations. If $\Phi_\beta(q,\ t,\ \theta,\ \omega)$ is a function of order β, homogeneous in t and

$$q=\sqrt{x^2+y^2+z^2}$$

and satisfying the wave equation

$$\frac{\partial^2\Phi_\beta}{\partial q^2}+\frac{2}{q}\frac{\partial\Phi_\beta}{\partial q}+\frac{1}{q^2}\frac{\partial^2\Phi_\beta}{\partial\omega^2}+\frac{\cos\omega}{q^2\sin\omega}\frac{\partial\Phi_\beta}{\partial\omega}+\frac{1}{q^2\sin^2\omega}\frac{\partial^2\Phi_\beta}{\partial\theta^2}-\frac{\partial^2\Phi_\beta}{\partial t^2}=0$$

then the function

$$\Phi=\frac{\Phi_\beta\,(q,\,t+\alpha(t^2-q^2),\,\theta,\,\omega)}{[1+2\alpha t+\alpha^2(t^2-q^2)]^{\beta+1}} \quad (2.4)$$

also satisfies this wave equation. It is interesting to note that the function η in (2.4) is identically equal to η in (2.3).

3. Let us consider a somewhat different problem. Let $\Phi_\beta(X,\ Y,\ T)$ be a function of degree β homogeneous in X, Y and T and satisfying the wave equation

$$\frac{\partial^2\Phi_\beta}{\partial X^2}+\frac{\partial^2\Phi_\beta}{\partial Y^2}-\frac{\partial^2\Phi_\beta}{\partial T^2}=0 \quad (3.1)$$

If

$$X = x + \alpha(t^2 - r^2),\quad Y = y,\quad T = t + \alpha(t^2 - r^2)$$

then we require to find $\eta\ (x,\ y,\ t)$ such that the function $\Phi = \eta\,(x,\ y,\ t)\ \Phi_\beta(X,\ Y,\ T)$ satisfies the wave equation (1.1).

This problem also has a solution. We find η from the following system of equations:

$$\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}+\frac{2\beta}{y}\frac{\partial\eta}{\partial y}-\frac{\partial^2\eta}{\partial t^2}=0,\quad \frac{\partial\eta}{\partial x}+\frac{t-x}{y}\frac{\partial\eta}{\partial y}+\frac{\partial\eta}{\partial t}=0 \quad (3.2)$$

$$(1-2\alpha x)\frac{\partial\eta}{\partial x}-\frac{1}{y}(X+2\alpha y^2)\frac{\partial\eta}{\partial y}-2\alpha t\frac{\partial\eta}{\partial t}-\alpha(2\beta+1)\eta=0$$

From the second equation of (3.2) it follows that η must depend on two variables only $\xi_1=(t-x)$ and $\zeta_1=x(t-x)-{}^1/_2y^2$. Introducing the new variables $\xi=\alpha(t-x)$ and $\zeta=\alpha^2(x(t-x)-{}^1/_2y^2)$, we use them to write the first and third equations (3.2) as follows:

$$(\xi^2-2\zeta)\frac{\partial^2\eta}{\partial\zeta^2}-2\xi\frac{\partial^2\eta}{\partial\zeta\partial\xi}-(2\beta+3)\frac{\partial\eta}{\partial\zeta}=0$$

$$(\xi+\xi^2-2\zeta)\frac{\partial\eta}{\partial\zeta}-(1+2\xi)\frac{\partial\eta}{\partial\xi}-(2\beta+1)\eta=0$$

Without going into an extensive analysis of the problem of obtaining a simultaneous solution of the above system of equations we note that the system admits a solution independent of ζ

$$\eta=C/(h+t-x)^{\beta+1/2}\qquad(h=1/2\alpha)$$

The wave equation is invariant under the rotation of the coordinate axes about the origin, therefore we can choose $x=r\cos(\gamma+\theta)$ and $y=r\sin(\gamma+\theta)$. Then

$$\Phi=\Phi_\beta\left(r\cos(\gamma+\theta)+\frac{1}{2h}(t^2-r^2),\,r\sin(\gamma+\theta),\right.$$
$$\left.t+\frac{1}{2h}(t^2-r^2)\right)\Big/[h+t-r\cos(\gamma+\theta)]^{\beta+1/2}\qquad(3.3)$$

will be a solution of the wave equation (1.1).

4. We shall use the simplest examples to illustrate the application of the results obtained to the problems of diffraction. We first note that the expression (1.9) enables us to use the solution of the plane wave diffracted by a wedge to find the solution of the problem of diffraction of the corresponding cylindrical wave. In exactly the same way the expressions (2.3) and (2.4) enable us to use the existing solutions of the problems of diffraction of the plane waves by the bodies consisting of semi-infinite straight lines and planes, to obtain solutions of the problems of diffraction of the spherical waves by the same bodies.

Let a unit plane wave $H(t-r\cos(\gamma+\theta))$ impinge on a semi-infinite plate containing the ray $\theta=\operatorname{arctg}(y/x)=0$. A solution of the problem of diffraction of this wave by the plate is given within the circle $r\leqslant t$ by the expression

$$\Phi_0=\Phi_0^++\Phi_0^-,\quad\Phi_0^\pm=\frac{1}{2}H(t-r\cos(\gamma\pm\theta))+\qquad(4.1)$$
$$\frac{1}{\pi}\operatorname{arctg}\left(\sqrt{\frac{2r}{t-r}}\sin\frac{\gamma\pm\theta}{2}\right)$$

Then the problem of diffraction for a cylindrical incident wave defined by the expression

$$\Phi=\sqrt{2R_0}H((R_0+t)^2-P_+^2)/[(2R_0+t)^2-r^2]^{1/2}$$
$$P_+^2=R_0^2+2R_0r\cos(\gamma+\theta)+r^2\quad(\alpha=1/2R_0)$$

where H is a unit function, is solved with the help of (1.9) within the circle $r\leqslant t$, and the potential is expressed by

$$\Phi=\Phi^++\Phi^-\qquad(4.2)$$
$$\Phi\pm=\frac{1}{2\sqrt{(2R_0+t)^2-r^2}}\Big[H((R_0+t)^2-P_\pm^2)+$$

$$\frac{2\sqrt{2R_0}}{\pi}\operatorname{arctg}\left(\sqrt{\frac{2r}{T-r}}\sin\frac{\gamma\pm\theta}{2}\right)\Bigg]$$

$$P_\pm^2 = R_0^2 + 2R_0 r\cos(\gamma\pm\theta) + r^2, \quad T = t + \frac{1}{2R_0}(t^2 - r^2)$$

For the second example we shall use the results of Sect. 3. If a plane wave $H(t - r\cos(\gamma+\theta))$ impinges on the same plate, then the corresponding incident wave which is also plane will be defined, by virtue of the expression (3.3), by the potential

$$\Phi = H(t - r\cos(\gamma+\theta)) \,/\, [h + t - r\cos(\gamma+\theta)]^{1/2} \qquad (4.3)$$

Returning to (4.1) we see that this expression is a sum of two functions $\Phi_0^+ + \Phi_0^-$, each of which satisfies the wave equation, and Φ_0^+ corresponds to the incident wave, while Φ_0^- corresponds to the reflected wave. We describe the diffraction of the wave defined by the potential (4.3) by forming, in the same way, two functions corresponding to the incident and the reflected waves

$$F\pm = \frac{1}{\sqrt{h+t-r\cos(\gamma\pm\theta)}}\left[\frac{1}{2}H(t - r\cos(\gamma\pm\theta)) + \frac{1}{\pi}\operatorname{arctg}\left(\sqrt{\frac{2P_\pm}{T-P_\pm}}\sin\frac{\Theta_\pm}{2}\right)\right] \qquad (4.4)$$

$$P_\pm = \left[r^2 + \frac{r}{h}(t^2 - r^2)\cos(\gamma\pm\theta) + \frac{1}{4h^2}(t^2 - r^2)^2\right]^{1/2}$$

$$\Theta_\pm = \operatorname{arctg}\frac{r\sin(\gamma\pm\theta)}{r\cos(\gamma\pm\theta) + (t^2 - r^2)/2h}, \quad T = t + \frac{1}{2h}(t^2 - r^2)$$

By virtue of (3.3) each of the functions F^+ and F^- satisfies the wave equation and their sum also satisfies the boundary conditions at $r = t$ as well as the condition of zero leakage at the plate.

Consequently the sum $F^+ + F^-$ is a solution of the problem of diffraction by a semi-infinite plate of a wave defined by the expression (4.3).

When h tends to zero, the second term in (4.4) within the square brackets tends to zero for $r < t$. Then the solution of the problem of diffraction by a semi-infinite plate within the circle $r < t$, of the wave defined by the potential

$$\Phi = H(t - r\cos(\gamma+\theta)) \,/\, [t - r\cos(\gamma+\theta)]^{1/2}$$

will be given by the following sum

$$\Phi = \Phi^+ + \Phi^-, \quad \Phi^\pm = {}^1/_2\,[t - r\cos(\gamma\pm\theta)]^{-1/2}.$$

In this case we follow [1], or use the expression (1.9) with $\beta = -{}^1/_2$, to obtain the solution of the problem of diffraction of a wave due to a plane source $(r < t)$ by a semi-infinite plate

$$\Phi = \Phi^+ + \Phi^-, \quad \Phi^\pm = \frac{1}{2}\left[(R_0 + t)^2 - R_0^2 - 2R_0 r\cos(\gamma\pm\theta) - r^2\right]^{-1/2}$$

provided that the incident wave is specified by the potential

$$\Phi = \frac{H((R_0 + t)^2 - R_0^2 - 2R_0 r\cos(\gamma+\theta) - r^2)}{[(R_0 + t)^2 - R_0^2 - 2R_0 r\cos(\gamma+\theta) - r^2]^{1/2}}$$

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MODEL OF AN ELASTIC PLATE OF FINITE ELEMENTS IN SUPERSONIC FLOW

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The dynamic stability of a thin plate in supersonic gas flow at low Strouhal numbers is examined. The aerodynamic forces are determined on the basis of the same partition mesh as for the representation of the plate as a model of finite elements. Rectangular elements with four coordinates at each node are used. The number of dynamical variables is diminished to one at each node as a result of reducing the order of the equation of motion. Examples of computing the plate vibrations in a vacuum and in fluid flow are presented.

Use of the finite-element method in aeroelasticity problems in the general case when the aerodynamic effects are determined numerically, is connected with great difficulties. This is related to the fact that a partition mesh not associated with the finite-element model representing the system is used to compute the aerodynamic forces. The aerodynamic mesh ordinarily consists of a comparatively large number of rhomboidal [1] or rectangular [2] cells and changes as the Mach number varies. The finite-element mesh has a larger spacing and is coupled rigidly to the structure. Changing it requires significant computational efforts.

It seems expedient to develop that approach to aeroelasticity problems in which the computation of the aerodynamic effects is performed on the basis of the same partition mesh as the description of the elastic-mass properties of the system. In order that an increase in the mesh spacing should not reduce the accuracy of the aerodynamic force calculation, the downwash within the limits of the cells is represented as a power series of the coordinates. The series coefficients are determined completely by the vector of the generalized coordinates of the element. Using the ordinary conjugate conditions of elements, the equation of motion of the elastic plate model in supersonic flow can be written in closed form without introducing a priori vibration modes

$$(K + \lambda^2 M)\mathbf{q} = \mathbf{Q}_a, \qquad \mathbf{Q}_a = \sum_{n=0}^{N} \lambda^n A_n \mathbf{q} \tag{0.1}$$